Pushing Information:
Realized Uncertainty and Notification Design*

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ABSTRACT

We study the dynamic information design problem of a firm seeking to influence consumer checking behavior under stochastic information arrivals. The firm's payoffs increase in the frequency of consumer checking. The consumer is uncertain about the arrival of information as well as its valuation. In addition to direct consumption utility, the consumer also has preferences over realized uncertainty: i.e., they experience disutility from the uncertainty of the information stock that has arrived, but which remains unchecked. This disutility increases in the variance of the information stock and can be interpreted as the anxiety costs resulting from unchecked information. The consumer's optimal checking behavior is a trade-off between costly checking and costly waiting. The firm can design a push notification mechanism to inform the consumer of the information arrival. We show that push notifications can lead to more frequent checking compared to no-push, even though it reduces the variance of the information. While push notifications resolve the information arrival uncertainty, they also create an endogenous impulse to check the information immediately. We consider generalized push strategies that allow the firm to add phantom notifications that do not contain valuable information. This noisy push strategy can lead to more frequent consumer checking, even though the consumer has rational expectations of the firm's strategy. We extend the model to account for consumer self-control and show that sophisticated consumers have the equilibrium incentive to block notifications. The main results also hold in a model with endogenous prices.

Keywords Information Design, Dynamic Persuasion, Push Notifications, Realized Uncertainty

JEL Codes D83, D91, L86, M31

I Introduction

With the proliferation of smartphones, tablets, and wearable devices, consumers are increasingly overwhelmed by information. With a few clicks or taps, consumers can check for the latest updates of their e-mail accounts, news feeds, and social media. The average American checks his/her phone every 12 minutes and about 80 times a day. A 2018 study by Kleiner Perkins Caulfield and Byers reports that the average amount of time spent by users on their smartphones has increased every year and now stands at 3.3 hours a day [1]. Recent studies have also documented the proliferation of information notifications that users face on their smartphones (Sahami Shirazi et al., 2014; Pielot et al., 2014).

An important reason for this growth in information checking and consumption is that in the modern digital economy, user attention (eyeballs) is a valuable commodity. Major firms such as Facebook and Google, as well as apps on the Android and Apple platforms, make money through user engagement, i.e., by inducing consumers to spend time and check their sites and apps as frequently as possible. Consequently, social media platforms and apps use strategic information design and presentation techniques to grab user attention and time. For example, it is a common practice in the industry to build the "variable rewards" paradigm into the app design: i.e., inducing users to stay longer and to check for more information by introducing uncertainty both into the timing and the value of future information arrivals. Twitter's spinning wheel indicates that the app is loading more content when the user swipes, but whether or not relevant and interesting content will be loaded is uncertain. Users who enable push notifications on Instagram will receive numerous messages from social connections including "stories" to attract them.[2]

Examining how the firm's notification technology influences consumer behavior and how that, in turn, influences the information design strategies of firms is important for understanding the efficiency and welfare consequences of these information markets. Numerous studies and industry reports indicate that consumers see themselves as being progressively addicted to smartphone usage and over-checking of information at the cost of personal pro-

[1] https://www.kleinerperkins.com/perspectives/internet-trends-report-2018/

[2] See Avery Hartmans "How app developers keep us addicted to our smartphones" *Business Insider*, 02/17/2018 for other examples.

ductivity (see Duke and Montag, 2017). Furthermore, recent studies show that smartphone users suffer anxiety/withdrawal costs if they are unable to check information notifications that they know have already arrived (Tams et al., 2018). Industry analysts also point to social media users experiencing what is labeled as the fear of missing out (FOMO), the idea that individuals want to constantly check for information because they worry that they will miss out on the news generated in their social network. Indeed, in response to growing concerns about the extent of addiction to devices and the over-consumption of information, both Apple and Google have recently developed digital health initiatives to help users to manage their information consumption on mobile devices.

We present a model of dynamic information consumption that captures the interaction between a strategic information provider and a forward-looking consumer who makes information checking decisions over time. Pieces of information arrive randomly over time according to a Poisson process, and they vary in their valuation (usefulness) to the consumer. For example, e-mails or updates on social media may arrive at different points in time. And they contain either relevant information or junk for the user. At each point in time, the consumer decides whether to check the information stock (e.g., unread e-mails in the inbox) by incurring some cost of checking.

The central aspect of consumer behavior that is captured in the model is preferences over "realized uncertainty." In other words, apart from the direct value of information consumption, the consumer also carries disutility for information that may have arrived but which they have left unresolved. Specifically, this disutility is increasing in the variance of the unresolved information stock. There are two different components of realized uncertainty in the information stock: about how many pieces of new information have arrived and about the valuation of each piece of information. This preference over realized uncertainty represents the previously described phenomenon: Consumers feel the compulsion to check their smartphones constantly due to the anxiety or the fear of missing out on potentially useful information. And the anxiety is higher when there is uncertainty in the number of information arrivals, and each piece of information has a higher variance.

The firm's problem is to choose an information (notification) design to maximize its profit, which is an increasing function of the frequency of consumer checking. Specifically, we consider the following commonly observed classes of notification design: At the one extreme, the firm can choose to adopt a passive "no-push" strategy in which the firm does not provide any notifications of information arrival. In this case, the consumer will make decisions based on the expectation of the information arrival process. The optimal strategy is to check at constant intervals of time. At the other extreme, the firm can use a genuine "push" strategy in which it commits to providing notifications to the consumer every time a new piece of information arrives. For example, the push notifications on iOS and Android for e-mail and other apps can alert consumers whenever a new piece of information arrives. The genuine push strategy reduces consumer uncertainty by resolving the uncertainty over how many pieces of new information have arrived at any point in time. In between these two extremes, there is a generalized "noisy push" strategy in which the firm chooses to add phantom push notifications even in the absence of true information arrival.

When will a push strategy be optimal for the firm? Within the framework, push notifications are likely to lead to more frequent checking compared to no-push when the variance in the value of the information is high relative to the mean. Under the no-push strategy, the consumer's anxiety costs and the impulse to check information increases in time. In other words, there is uncertainty both in the arrivals and the value of information. Given the Poisson process, the total uncertainty increases linearly in time. The genuine push strategy fully resolves the arrival uncertainty. But every time there is a notification, the consumer also faces realized uncertainty in that she knows that there is a piece of information whose valuation is unknown that is waiting to be consumed. This increases the anxiety costs of waiting discontinuously. Consequently, there is an endogenous impulse to check the information immediately to nullify the jump in anxiety cost, which would result from carrying the realized uncertainty. And this impulse is increasing in the variance of the information value. If the impulse is strong enough (when the value of the information varies a lot), the consumer may check at every m push notifications, which may be more frequent than in the no-push case. In other words, push notifications can induce a spread in the consumer's belief such that she either believes that there's no information, or enough amount of information to trigger her checking behavior that she otherwise would not do. Push notifications by definition lead to a discretization of checking which can locally accelerate the consumer's checking behavior depending upon the evolution of beliefs. The push strategy can also be a Pareto improvement because it leads to higher profits for the firm through increased checking and higher consumer utility through reduced uncertainty.

We then consider the case of noisy push whereby even in the absence of genuine information arrival, the firm strategically adds phantom pushes with no informational value for consumers. This strategy represents the motivation of platforms to induce more consumer checking to monetize consumer attention. We characterize the optimal noisy push strategy and show that the equilibrium amount of noise, in general, falls in the interior between the extremes of no-push and genuine push. In equilibrium, the noisy push strategy can generate higher profits for the firm compared with the genuine push strategy. The firm wants to increase phantom pushes as much as it can get away with. But the rational consumer expects this and will have the incentive to reduce the frequency of checking. The firm's optimal choice balances these forces such that the consumer is just indifferent between checking at every m notifications (which was optimal for the genuine push case) and every $m+1$ notifications. The incentive to add phantom pushes increases if there is higher anxiety, lower information arrivals, higher information variance, and lower mean consumption value.

We extend the model to account for consumer self-control problems to explain the prevalence of consumer blocking of notifications. We develop a dual-self model, in which the long-term self has lower disutility over realized uncertainty compared with her short-term self. The long-term self can influence the short-term self's checking behavior by accepting or blocking notifications. We find that the long-term self may block notifications if push notifications lead to a higher checking frequency. We also extend the model to endogenous prices by allowing the platform to charge consumers a subscription fee on top of profits from consumer checking. The firm prefers the genuine push strategy if it leads to more frequent checking because the subscription fee will then be also higher under the push strategy. Comparing the noisy push with the genuine push strategy, the firm faces the trade-off of more profits from checking due to more push notifications, and a higher subscription fee due to less noise. Therefore, we find that its optimal noise level is the same as in the main model if checking were to be socially optimal, and lower if checking is not socially optimal yet still large enough.

II Related Research

The literature on information disclosure (e.g. Grossman, 1981; Milgrom, 1981) usually assumes that the sender's information disclosed to the receivers is conditional on their private information. This leads to the standard unraveling result that all quality types are separated and disclosed in equilibrium.[3] In our context, we focus on the problem of a firm pre-determining the policy of notifications prior to the information arrivals. In other words, the firm is able to commit to an information policy before the state of the world is realized. Therefore, the problem we study is related to the information design / Bayesian persuasion literature (e.g. Kamenica and Gentzkow, 2011; Bergemann and Morris, 2019; Kamenica, 2018). Specifically, this paper is most closely related to the problem of dynamic Bayesian persuasion (Ely, 2017). Ely (2017) focuses on characterizing the general dynamic persuasion mechanisms and considers the problem of a sender who seeks to persuade a myopic receiver who takes action in each period based only on the current state. In contrast, in this paper, the sender faces the dynamic persuasion problem of a consumer who is strategic and forward-looking. Consumers know that their current decision to check will dynamically affect their actions in future periods and this makes the anxiety thresholds endogenous to the firm's information policy. In addition, our paper also characterizes a demand-side foundation that rationalizes the consumers' information checking impulse and its implications for the firm's strategy.

Our model characterizes a psychological micro-foundation of disutility for realized uncertainty. At any point in time, consumers have a disutility for the stock information which might have arrived but is unresolved, and this disutility is proportional to the variance of the information stock. This characterization of realized uncertainty ties together interpretations from several strands of research in psychology. First, it can be interpreted as the anxiety arising from intolerance to uncertainty (e.g. Ladouceur et al., 2000). Recent studies in experimental economics show subjects prefer early resolution of uncertainty, even if the information is not instrumental (Falk and Zimmermann, 2016; Ganguly and Tasoff,

[3] The subsequent literature has proposed several ways to mitigate this information unraveling problem. These include uncertainty in information endowment (Dye, 1985) and costly communication (Jovanovic, 1982; Verrecchia, 1983; Guo and Zhao, 2009; Lu and Shin, 2018).

2017; Masatlioglu et al., 2017; Nielsen, 2017). Closely related is the phenomenon of the preference for indeterminacy: For example, (Brun and Teigen, 1990) show that subjects prefer to guess the outcome of a coin toss before rather than after the event occurs. Similarly, in the actual context of sports consumption (Vosgerau et al., 2006) consumers show preferences for indeterminacy and prefer live TV content compared to recorded. Our formulation provides a way to represent intolerance to uncertainty or the preference for indeterminacy. Second, the desire to resolve realized uncertainty can also be interpreted as the curiosity impulse (Loewenstein, 1994) or the "information-gap" theory, which argues that individuals will become more motivated to know something if they learn that it is knowable.

More generally, this research is also related to the literature on belief-based utility: For example, Kőszegi and Rabin (2009) present a model in which agents are loss averse over be over changes in beliefs about present and future consumption. Closer to our analysis is Ely et al. (2015) who study the optimal information revelation plan of a principal when the agent has non-instrumental information preference that involves entertainment utility over suspense (when the next period's beliefs have greater variance) versus surprise (when the current beliefs are further away from the last period's beliefs).

III The Model

We develop a model of an information market to characterize the dynamic interaction between a consumer and an information provider. The model captures many important information consumption contexts that consumers regularly encounter, including news consumption, email usage, social-network updates, and instant messaging. The common feature characterizing these information consumption contexts is that information of varying usefulness can arrive over time, and the information provider (firm) can strategically design the format in which the information is presented to the consumer. For example, news information is generated exogenously and arrives over time, but the news provider can choose how to organize, aggregate, and transmit the information. We start by first specifying the information arrival process and its valuation.

A Information arrival

Nature generates information, and pieces of information arrive according to a Poisson process with a rate of $\lambda > 0$. Therefore, the number of information arrivals N in a time interval τ is given by $P(N(\tau) = k) = \dfrac{e^{-\lambda\tau}(\lambda\tau)^k}{k!}$, $k \in \mathcal{N}$, and the expected number of arrivals for time τ is given by $\mathrm{E}[N(\tau)] = \mathrm{Var}[N(\tau)] = \lambda\tau$. Furthermore, a piece of information varies in its usefulness. For example, an email waiting in the consumer's inbox may contain an invitation for a job interview, or alternatively may be a generic promotion message in which the consumer has no interest. To capture this feature, we assume that the k-th piece of information yields the consumer a random utility I_k, which is i.i.d. with mean $\mathrm{E}[I_k] = E_I$ and variance $\mathrm{Var}[I_k] = V_I$, and is independent from the information arrival process. Thus, the consumer has uncertainty about both the timing of information arrivals and the value of each piece of information. Next, we specify the consumer behavior and the information consumption utility.

B Consumer

Consider a consumer (female), who chooses between adopting information provider's service or an outside option. If she adopts the service, she will have access to the information that arrives through the process described in the previous section. At any point in time, the consumer can choose to consume the information stock that has accumulated until that point by deciding to check for information. Alternatively, if she does not check, the information will keep accumulating according to the Poisson process and will be available for potential future consumption. The consumer will incur a checking cost every time she checks, which is normalized to one without loss of generality.

The consumer utility function has two components: First, the consumer gets utility from information consumption. The consumption utility of checking a piece of information is given by the random variable I_k specified above. If the consumer checks at some point in time, the total consumption utility is given by $u = \sum_{j \in J} I_j$, in which J is the set of information stock at the time of checking. Second and central to the theory of this paper, the consumer also experiences a disutility from leaving unchecked the information that might have already arrived. This disutility from realized uncertainty is what represents the consumer's anxiety cost or compulsion to check information. We assume that it is given by a flow disutility $u_t' = -\rho \text{Var}[u_t]$, in which u_t is a random variable capturing the consumption utility that could have been realized if the consumer checks at time t, and $\rho > 0$ is a parameter capturing the magnitude.[4] The disutility has several interpretations that are relevant for the information consumption contexts that motivate this analysis: First, it captures the anxiety cost resulting from uncertainty of not consuming useful information (Freeston et al., 1994; Ladouceur et al., 2000). The idea that uncertainty can induce anxiety is documented in psychology. For example, Tolin et al. (2003) show that checking compulsions are correlated with intolerance of uncertainty. We emphasize that it is the *realized* uncertainty, which could have been resolved by the consumer's actions, that causes the disutility. In other words, anxiety arises precisely because the consumer had the agency to resolve the uncertainty but chose not to. Second, it is also consistent with the psychological literature on curiosity and the information-gap mechanism (Loewenstein, 1994).

C Firm (Information Provider)

The firm profits from the consumer's checking behavior. In the main model, we assume that its profit is an increasing function of consumer checking. Therefore, the firm maximizes consumer checking frequency. Later, we extend the analysis by allowing the firm to charge the consumer a subscription fee of adopting the service.

The firm's strategy is to choose an information design to influence consumer choice. It can choose from a rich set of information policies to influence consumer checking behavior. As in the Bayesian persuasion literature the firm commits ex-ante to an information policy, and the consumer reacts rationally. We consider the following commonly observed information policies. The firm can choose to provide the consumer with access to information

[4] Note that we require that ρ is strictly positive as the theory in this paper is about how preferences over realized uncertainty affects information consumption and notification design. If $\rho = 0$ and there is no anxiety caused by realized uncertainty, then the consumer would not check in a finite time. An alternative reason for the consumer to check would be the depreciation of information value over time. But in this paper, we focus on the role of preferences over realized uncertainty and anxiety costs.

passively. This is the *No-Push Strategy*, where the firm does not provide any notification. Alternatively, the firm can organize and notify the consumer, and at the extreme, it can commit to a *Genuine Push Strategy*, where the firm commits to pushing a notification to the consumer when and only when there is an information arrival. In this case, the consumer learns about the arrival of information immediately and then decides whether or not to incur the cost to check the information stock and resolve the valuation uncertainty.

However, given that the firm's objective function is increasing in the amount of checking, a more general information design problem for the firm would be to add notifications over and above the true information arrivals. In other words, the firm can adopt a *Noisy Push Strategy* in which it allows the addition of "phantom" push notifications even in the absence of actual information arrivals. These phantom pushes are empty, and therefore they have zero informational utility for the consumer. To characterize this in the information market, assume that in addition to the true information arrival process, there also exist additional arrival processes which have zero informational value. The firm chooses whether and how many of the phantom notifications to add. For example, Facebook may allow push notifications which end up being irrelevant updates with no informational value for the consumer. Assume that the arrival of phantom pushes is a Poisson process with a rate $k\lambda$ that is independent of the true generic information arrival process. The composite push notification process, which the consumer observes, is also Poisson with rate $(k+1)\lambda$. When $k \to \infty$, this converges to the no-push case, and when $k \to 0$ it converges to the genuine push notification case. Note that while the noisy push strategy with the additional phantom pushes can benefit the firm because it may induce more frequent checking, the rational inference of the presence of phantom pushes on the part of the consumer will also induce her to pull back on the checking behavior.

IV Analysis: Commitment to Truthful Notifications

To highlight the mechanisms that influence information consumption, we start with the baseline in which the firm chooses between a genuine push strategy and a no-push strategy. We start the analysis by characterizing the consumer's optimal information consumption behavior given the firm strategy and then the firm's equilibrium choice of the information design.

A Optimal Consumer Checking Behavior

The following lemma prescribes consumer behavior in the case of the no-push strategy.

Lemma 1 *Without push notification, the consumer's optimal checking behavior is to check every t_n^* units of time, in which $t_n^* = \sqrt{\frac{2}{\rho(V_I + E_I^2)\lambda}}$.*

Proof. See Appendix A. ■

Without any push notifications, the consumer's optimal checking strategy is stationary and involves a constant interval between consecutive checks. The consumer makes a trade-off between the cost of checking and the cost of waiting. The cost of waiting is the flow disutility of realized uncertainty, which consists of two components: how many pieces of information have arrived, and how useful is each piece of information. Both increase linearly in the time since the last check. Therefore, the opportunity cost of forgoing checking also increases in time. The first component is proportional to the mean utility E_I, and the second is captured by the variance of information utility V_I. The anxiety cost is also proportional to the information arrival rate λ and anxiety parameter ρ, and so as expected, the consumer checks more often as these parameters increase.

Consider now the genuine (truthful) push strategy: In this case, the consumer's checking behavior is contingent on the arrival of observed push notifications. We will show in the analysis below that the consumer has no incentive to check when there is no notification, no matter how long she has waited since the last check. The consumer also has no incentive to check between two consecutive notifications. This is because that strategy is dominated by either checking at the previous notification or the subsequent one. The optimal checking behavior is therefore characterized by checking every n_p^* notifications, which can be specified as follows:

Lemma 2 *Define $\tilde{n}_p = \arg\min_{n \in \mathcal{R}_+} \frac{\lambda}{n} + \rho \frac{(n-1)}{2} V_I = \sqrt{\frac{2\lambda}{\rho V_I}}$. Then the consumer's optimal checking behavior with genuine push notification is to check at every n_p^* notifications which is as follows:*

a. $n_p^* = \lfloor \tilde{n}_p \rfloor$, *if* $\tilde{n}_p \leq \sqrt{\lfloor \tilde{n}_p \rfloor (\lfloor \tilde{n}_p \rfloor + 1)}$.

b. $n_p^* = \lceil \tilde{n}_p \rceil = \lfloor \tilde{n}_p \rfloor + 1$, *if otherwise.*

Proof. See Appendix B. ■

The optimal checking behavior of the consumer trades off the the checking cost $(\frac{\lambda}{n})$ which is increasing in the checking frequency with the the waiting cost $(\rho \frac{(n-1)}{2} V_I)$ that is decreasing in the checking frequency. Minimizing the combination of these costs and ignoring the integer constraint gives us $\tilde{n}_p = \sqrt{\frac{2\lambda}{\rho V_I}}$. In general this $\tilde{n}_p$ will lie in the interval $[\lfloor \tilde{n}_p \rfloor, \lfloor \tilde{n}_p \rfloor + 1]$, where $\lfloor \tilde{n}_p \rfloor$ denotes the floor of $\tilde{n}_p$.

In the presence of push notifications the consumer's checking space is restricted from $\mathcal{R}_+$ to $\mathcal{N}_+$. In other words, push notifications imply a constraint on the checking choices. Notifications are by definition discrete and so the consumer will have to choose between checking every $\lfloor \tilde{n}_p \rfloor$ or $\lfloor \tilde{n}_p \rfloor + 1$ notifications. Therefore, there are two countervailing effects that notifications have on consumer behavior (compared to no-push). On the one hand, push notifications from the firm unambiguously reduce realized uncertainty. Under the no-push strategy, the consumer is uncertain about i) how many pieces of information have arrived and ii) their value. Genuine push notifications resolve the uncertainty of the first component by always informing the consumer of the exact number of information arrivals at every point in time. On average, the consumer suffers less from leaving any information unchecked. Therefore, one may expect the genuine push strategy to reduce the incentive to check. However, notifications have a second potentially countervailing effect, namely, they constrain the consumer choice space, and this can induce faster checking. The condition in part (a) of the Lemma provides insight into when that might be the case.

When $\tilde{n}_p \leq \sqrt{\lfloor \tilde{n}_p \rfloor (\lfloor \tilde{n}_p \rfloor + 1)}$, the consumer will choose to check every $\lfloor \tilde{n}_p \rfloor$ rather than waiting for one more notification to arrive. In other words, the consumer's checking behavior is *locally accelerated* and the consumer is induced to check more frequently. This is achieved by how the notifications change the evolution of the consumer's beliefs. Under no-push, the belief about the number of arrivals is increasing linearly in time. This belief is not strong enough to trigger checking until $t = t_n^*$. But with push notifications, while the mean belief is the same at every point in time (the consumer is Bayesian), the belief is spread between either zero information arrival when there is no notification, or the exact number of information arrivals. When the number of arrivals is large enough, the local acceleration mechanism kicks in, inducing the consumer to check.

Specifically, smaller the information arrival rate λ and higher the anxiety costs ρV_I the greater is possibility that the consumer accelerates her checking and chooses to check every $\lfloor \tilde{n}_p \rfloor$ notifications rather than waiting for an additional notification. In contrast, when $\tilde{n}_p > \sqrt{\lfloor \tilde{n}_p \rfloor (\lfloor \tilde{n}_p \rfloor + 1)}$, the consumer slows down her checking frequency by waiting for an additional notification.

The natural question that we can now ask is whether genuine push notifications can lead to increased checking compared to the no-push strategy, despite the fact that they reduce the amount of realized uncertainty for the consumer. We proceed to investigate this question and the resulting implications for the firm.

B Optimal Firm Strategy: To Push or Not?

Comparing the case with and without push notifications, we can identify if there exist conditions under which we get the result that push notifications can lead to more frequent checking despite lowering the consumer's uncertainty. The following proposition provides the comparison:

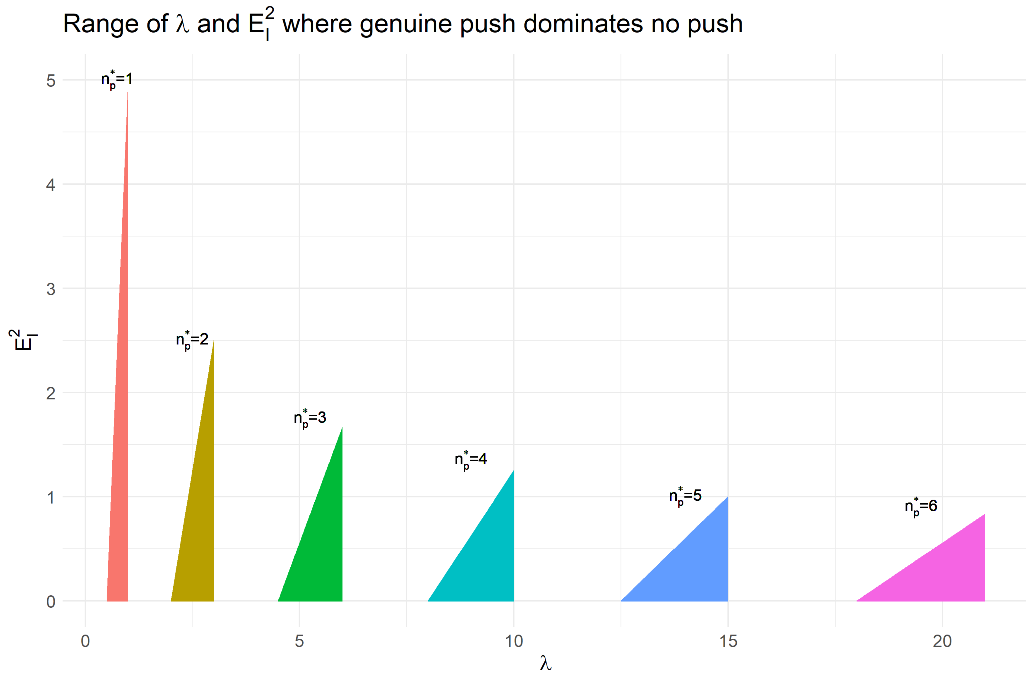
Proposition 1 *If $V_I > \lfloor \tilde{n}_p \rfloor (E_I)^2, \lfloor \tilde{n}_p \rfloor \in \mathcal{N}_+$, there exists a parameter range such that the genuine push strategy leads to higher checking frequency. This range is given by:*

$$\lambda \in \left(\rho(V_I + E_I^2) \frac{\lfloor \tilde{n}_p \rfloor^2}{2}, \rho V_I \frac{\lfloor \tilde{n}_p \rfloor (\lfloor \tilde{n}_p \rfloor + 1)}{2} \right). \tag{1}$$

In this range, the consumer optimally checks at every $n_p^* = \lfloor \tilde{n}_p \rfloor$ push notifications, and the firm's equilibrium strategy is to use the genuine push strategy over no-push. Otherwise, the firm adopts the no-push strategy.

Proof. See Appendix C. ■

Why does push notification lead to more checking? Under the no-push strategy, the consumer's anxiety cost increases linearly in time: Both the uncertainty about the number of information arrivals and the uncertainty in their valuations increase linearly in time. And

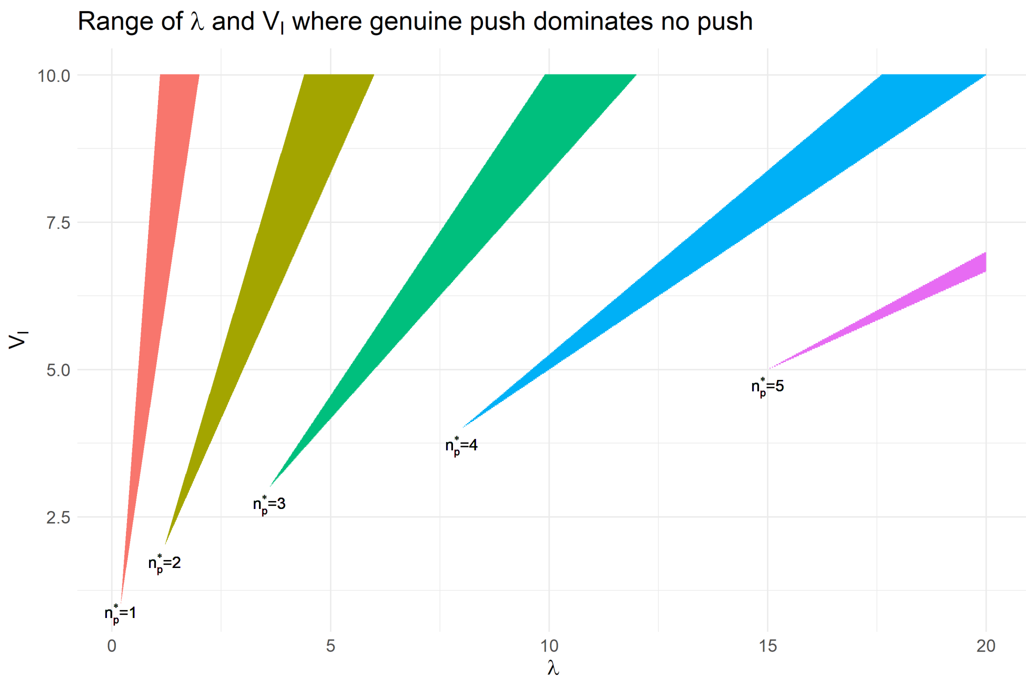


Note: Parameter ranges of λ and E_I^2 in which the push strategy leads to higher checking frequency, given the parameters of $\rho = 0.2$, $V_I = 5$.

Figure 1: Parameter ranges in which genuine push dominates no-push

when the overall anxiety cost reaches a certain threshold, the consumer "pulls the trigger" and incurs the checking cost to nullify the built-up stock of anxiety.

With push notifications, the consumer's anxiety cost level becomes a step function in time. Whenever there is a notification, the anxiety cost increases discontinuously. This is because the consumer now knows for sure that there is a new piece of information that has arrived and it has the associated variance of V_I. Therefore, the consumer may be impelled to check because the cost of waiting increases discontinuously. Next, recall that push notifications endogenously constrain the consumer's checking choices from $\mathcal{R}_+$ to $\mathcal{N}_+$. If she does not check now, she will have to wait until the next information arrival, because checking between two notifications is never optimal. Therefore, depending upon the characteristics of the information arrival and its valuation, it is possible that push notifications can lead to accelerated checking. We discuss this below.



Note: Parameter ranges of λ and V_I in which the push strategy leads to higher checking frequency, given the parameters of $\rho = 0.2$, $E_I^2 = 1$.

Figure 2: Parameter ranges in which genuine push dominates no-push

The strength of the impulse to check depends on the variance of information value V_I. When it is large enough (relative to the mean utility E_I), the consumer may check more often under genuine push notifications. This implies the interesting point that even though, on average, the push strategy leads lower expected uncertainty and anxiety costs, the consumer might still end up checking more often with push compared to the no-push strategy. Conversely, when V_I is small relative to E_I, the consumer checks less often under the push notification strategy. Therefore, the firm's optimal decision to use push notification or not depends on the relative value of V_I.

The condition (1) highlights the mechanisms which lead push notifications to encourage more frequent checking. The firm is to uses push strategies when the arrival rate (λ) is given the interval in (1). The arrival rate λ is required to be neither too low nor too high relative to the anxiety level. Consider the case when λ is low and below the lower boundary of the interval, i.e., $\rho(V_I + E_I^2)\frac{\lfloor \tilde{n}_p \rfloor^2}{2}$: Information arrives infrequently, and the number of realized notifications will be relatively low. But under no-push, the consumer's checking given the anxiety costs would still be higher (with some of those checks resulting in no incremental information). But when λ is sufficiently high, information arrives frequently. The consumer will likely check after the collection of more pieces of information. In this case, suppose the consumer were to forego checking at some n number of notifications and decides to wait for an additional notification to arrive. She knows that the wait for the next notification is not likely to be very long. This alleviates the anxiety cost and leads to less frequent checking.

Figure 1 represents the condition (1) and shows the range of λ and E_I^2 for which push notifications dominate no-push for the firm. We can see that for any n_p^* a lower E_I^2 expands the range over which the genuine push strategy is chosen by the firm. In other words, when the expected value of the information is lower, the consumer waits longer to check autonomously under no-push. Therefore the firm is more inclined to choose push notifications. Figure 1 shows that when E_I^2 is small, there are more cases of n_p^* and larger ranges of λ for which the push notification strategy dominates. Next, for any given E_I^2 and n_p^*, there is an upper boundary of the arrival rate $(\rho V_I \frac{n_p^*(n_p^*+1)}{2})$ below which the consumer will choose to accelerate her checking to every n_p^* rather than waiting for one more notification. It is this accelerated checking that induces the firm to prefer the push notification strategy. When the arrival rate further increases, the consumer will wait to check every $n_p^* + 1$ notifications, and this induces the firm to prefer the no-push strategy.

We can also see that as arrival rate increases (as does n_p^*), the gap between any two intervals also increases. In other words, as information arrives more frequently, we have that the no-push strategy is chosen over a greater range of market conditions. Indeed as $\lambda \to \infty$ no-push is always preferred by the firm. Next, Figure 2 shows the range of λ and V_I^2 for which push notifications dominate no-push for the firm. The observations are similar: First, for any n_p^*, a higher variance expands the range of λ for which the push strategy is preferred by the firm. Second, when the variance is large, there are more cases of n_p^* that the push strategy could dominate.

Having characterized the optimal firm strategy, we move on to analyze the consumer surplus under push notifications:

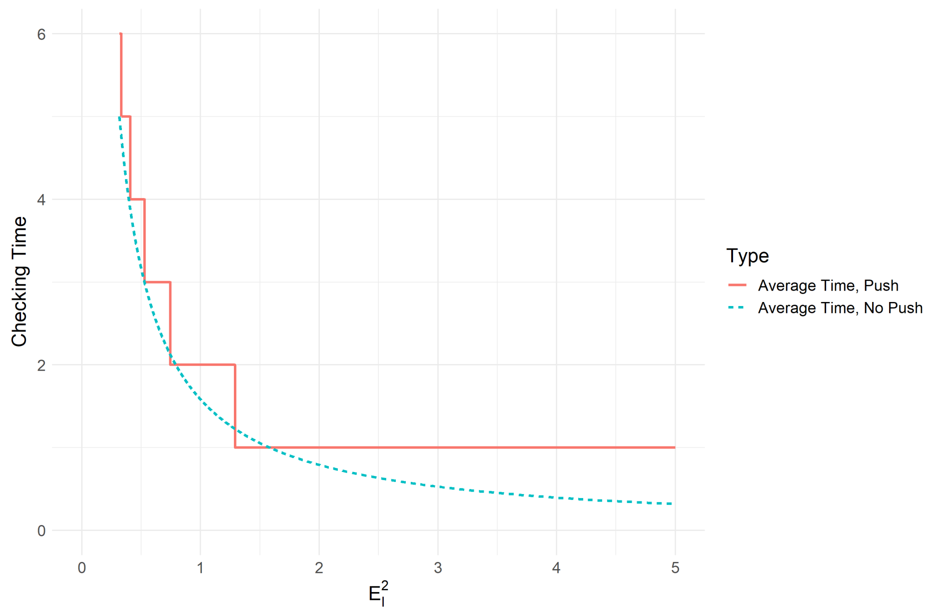
Corollary 1 *The consumer's expected utility is higher under the genuine push as compared to no-push. Therefore, if push notifications lead to more frequent checking, it is a Pareto improvement over the no-push strategy.*

Proof. See Appendix D. ■

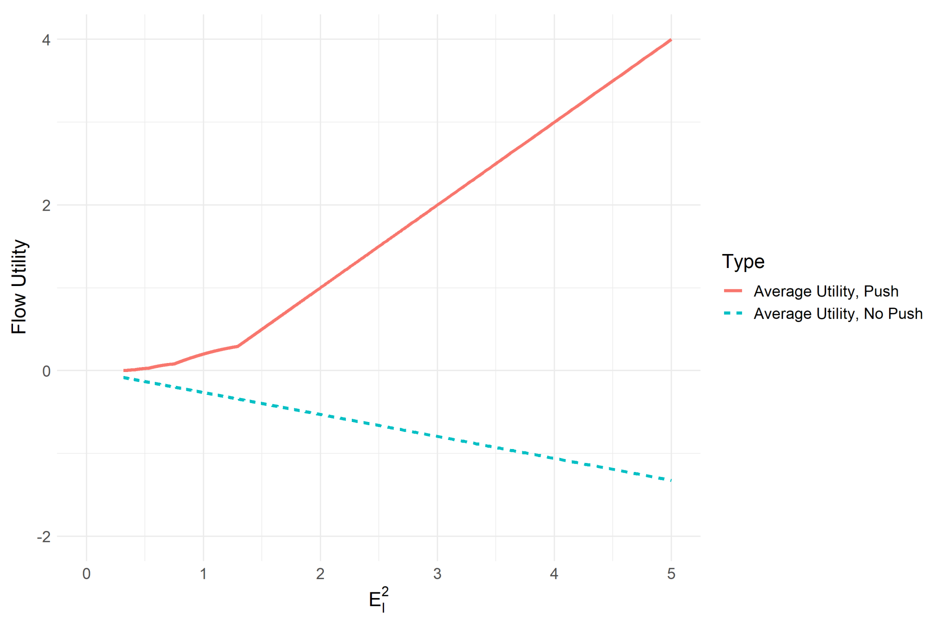
The consumer's utility is higher under the genuine push because the anxiety cost is lower due to less uncertainty if she adopts the same checking strategy under no-push. Thus, her expected utility under the optimal checking strategy will be even higher. Corollary 1 highlights that push notification can be a Pareto improvement that benefits both the consumer and the firm when the variance of the information is sufficiently large. In that case, compared with no-push, the push strategy is socially optimal. As a numerical example, Figure 3a illustrates, the average checking time for push and no-push strategies using the following parameters: $\lambda = 1$, $\rho = 0.2$, $V_I = 3(E_I)^2$. In the range of $E_I = (1.29, 1.58)$, the consumer checks more frequently with push strategy. Figure 3b shows the consumer expected utility for the same parameter range. We observe that the consumer utility under genuine push is higher than without push notifications, as Corollary 1 suggests.

V Noisy Push Strategy

Up until now, we have restricted the analysis to the case in which the firm is committed to a truthful information design policy of providing genuine notifications. We now consider the case in which the firm has the ability to add phantom push notifications that do not contain any useful information for the consumer. Given that the firm's payoffs are increasing in consumer checking, the firm may strategically adopt a noisy push strategy that could motivate increased checking by mixing genuine notifications with phantom ones which do not contain any useful information. This may be the case even if the consumer has rational expectations of the firm's strategic behavior. Suppose that the firm can add phantom pushes at a rate of $k\lambda$ that is independent of the true information arrival and let us label



(a) Average checking time



(b) Consumer utility

Note: Numerical example of average checking time and utility for no-push and genuine push strategies, parameters given by $\lambda = 1$, $\rho = 0.2$, $V_I = 3(E_I)^2$.

Figure 3: Numerical example of checking time and utility

this as a k-noisy push process.[5] Under the noisy push strategy, the rational consumer will be skeptical and less motivated to check any given notification. The question is whether it is still possible that they may check more in the aggregate? When will noisy push be an equilibrium for the firm? The following proposition describes the equilibrium.

Proposition 2 *Define* $\tilde{n}_x = \arg\min_{n \in \mathcal{R}_+} \frac{(k+1)\lambda}{n} + \frac{\rho(n-1)}{2(k+1)}(V_I + E_I^2 \frac{k}{k+1})$, *and* $n_x^* = \lfloor \tilde{n}_x \rfloor$. *The following characterizes the noisy push equilibrium.*

- *When* $V_I \geq E_I^2 \frac{n_x^* - k^*}{1+k^*}$ *the firm's equilibrium strategy will be to use a noisy push strategy compared with both the genuine push strategy and the no-push strategy.*
- *In equilibrium, the consumer will check every* n_x^* *notifications.*
- *The firm's optimal phantom push intensity* k^* *under the noisy push strategy is given by the following implicit equation,*

$$\frac{(k^*+1)^2\lambda}{n_x^*(n_x^*+1)} = \frac{\rho}{2}V_I + \frac{k^*}{2(k^*+1)}E_I^2 \tag{2}$$

Proof. See Appendix E ■

As in the case of the basic model in Lemma 2, the consumer's optimal checking behavior minimizes the checking and waiting cost up to the integer constraint. In the case of a k-noisy push process, with every notification, the consumer will infer that the probability of genuine information arrival is $\frac{1}{1+k}$. As before, the consumer's anxiety cost will increase discontinuously at every notification but now to a lesser extent than in the case of genuine push (and it will not vary between any two notifications). So the consumer's optimal checking behavior is to check at every n_x^* notifications, which given the consumer's rationality, is weakly larger than with genuine push. When $k = 0$, the checking behavior is identical to the checking behavior under genuine push reported in Lemma 2.

Despite the fact that the consumer fully anticipates the presence of phantom pushes, it is nevertheless possible for the firm to gain by adding phantom pushes and to induce the consumer to check more. Note that in equilibrium the firm will optimally choose the amount

[5] Alternatively, suppose that the firm had access to a continuum of possible phantom information processes with Poisson arrival rate $k\lambda$ which is independent of the true information arrival and could choose k.

of noise k^* such that the consumer will prefer to check every $n_x^* = \lfloor \tilde{n}_x \rfloor$ rather than waiting for an additional notification. Given this $V_I \geq E_I^2 \frac{n_x^* - k^*}{1 + k^*}$ then represents the condition when the noisy push payoffs are greater than the the case of no-push. The proposition also makes precise the optimal amount of noise the firm can add through phantom pushes. The firm wants to increase phantom pushes as much as it can get away with. However, the rational consumer expects that the probability of arrival to be $\frac{1}{1+k}$ $\forall k$ and would want to (weakly) reduce the frequency of checking. The firm's optimal choice balances these forces by adding phantom pushes $k = k^*$ such that the consumer is just indifferent between checking at every $\lfloor \tilde{n}_x \rfloor$ notification and every $\lfloor \tilde{n}_x \rfloor + 1$ notifications.

Notice that k^* is determined by the condition Equation 2 which is an implicit equation whose closed-form solution is complicated. But we can still characterize the comparative statics: $\frac{\partial k^*}{\partial \rho} > 0$, $\frac{\partial k^*}{\partial \lambda} < 0$, $\frac{\partial k^*}{\partial V_I} > 0$, and $\frac{\partial k^*}{\partial E_I} < 0$. The firm in equilibrium adds more phantom pushes when ρ is higher. In other words, the firm responds to consumer anxiety by adding more phantom pushes. Further, lower information arrival rates allow the firm to add more noise – i.e., when the true information arrival is more infrequent, the firm can maintain consumer checking behavior with even higher levels of noise. Also, a higher variance and a lower mean of the information consumption utility, as expected, lead the firm to add more phantom pushes.

In a special case when $E_I \to 0$, we have a stronger result that the noisy push will dominate no-push for any parameter range of λ and V_I.

Proposition 3 *When $E_I \to 0$, the optimal noisy push will lead to more frequent checking than no-push. Let m given by $\lambda \in \left[\rho V_I \frac{m(m-1)}{2}, \rho V_I \frac{m(m+1)}{2}\right)$. The average time per check under noisy push is $t_p^* = \frac{m}{(k^*+1)\lambda} = \sqrt{\frac{2m}{\rho V_I \lambda (m+1)}} = t_n^* \sqrt{\frac{m}{m+1}}$, where t_n^* is the average time per check under no-push as in Lemma 1.*

Proof. See Appendix G ■

Proposition 3 shows that when $E_I \to 0$, the noisy push strategy can consistently induce more frequent checking compared to the no-push strategy. Firms will have the incentive to use noisy push precisely when the information on average has less value. In fact, in this case, the noisy push strategy will increase the checking frequency to $\sqrt{\frac{m+1}{m}}$

where m is the number of notifications that the consumer checks. For example, when V_I is high such that $m = 1$, i.e., the consumer checks at every notification, the noisy push strategy can improve checking frequency to $\sqrt{2}$, or by 41%, relative to the no-push strategy.

Compared with the no-push strategy, the noisy push strategy could increase consumer surplus, because it still gives the consumer some information to alleviate the anxiety cost of uncertainty. Similar to the case in the genuine push strategy, the consumer will enjoy a higher surplus compared with the no-push strategy.

Corollary 2 *In equilibrium, the noisy push strategy with k^* leads to higher consumer surplus compared with no-push strategy.*

Proof. See Appendix F ■

VI Self-Control Problems and Consumer Blocking

The model that we have analyzed thus far is one with a consumer who has time-consistent preferences over the realized uncertainty: i.e., at any time t the consumer has the same anxiety parameter ρ for the current period t anxiety and as well the expected future anxiety (evaluated at time t).

However, the information consumption situations that motivate this paper may naturally involve the tension between short-term impulses and long-run preferences. The consumer may be subject to self control problems and have the preference to check more frequently than what their long term selves might wish to do. The basic model cannot capture the self-control problem that would lead to such information addiction. In our setup, preferences for realized uncertainty predicts that (rational) the consumer will always be better off under push notification compared with no-push, even when the notifications may be noisy. However, we commonly observe information consumption situations where consumers have the tendency to actively block notifications. The model described below links consumer self-control problems to their blocking behavior and then investigates the effect on firm incentives.[6]

[6]Thus the analysis in this section adds to research stream that investigates the responses of firms to

Considering self control problems provides a rationale for why the consumer may strategically block notifications even when they reduce the overall variance in the realized information. To capture the self-control problem in information consumption in a parsimonious manner, we extend the basic model by adopting a dual-self framework (e.g. Thaler and Shefrin, 1981; Fudenberg and Levine, 2006). Accordingly, assume that there is a long-term self and a short-term self. The short-term self is endowed with the same preference as in our basic model. However, the long-term self has lower anxiety over future realized uncertainties and evaluates these uncertainties with parameter $\hat{\rho} < \rho$. Another interpretation of this setup is that the long-term self is subject to less temptation (of checking), but otherwise has the same preference as the short-term self. The long-term self is also sophisticated. She correctly predicts that the short-term self will decide when to check under parameter ρ.

The timeline is as follows. At the beginning $T = 0$, the firm makes both push and no-push strategies available. The long-term self chooses an information policy, i.e., push technology, for the short-term self. For example, it may happen at the point when the consumer installs an app or the first time the operating system asks whether the consumer wants to block push notifications from the app, or whether the consumer wants a pre-set notification frequency. After this initial decision by the long-term self, the short-term self then chooses when to check for information in real-time in every consumption period conditional on the information policy. The decisions of the consumer therefore become an intra-personal game between the long-term self's decision at $T = 0$ and the short-term self who makes decisions in every consumption period.

The following proposition characterizes the result:

Proposition 4 *If the push notification strategy leads to more frequent checking than no-push, there exists a threshold* $\hat{\rho^*}$, *such that the long-term self will choose to block notifications by committing to no-push if* $\hat{\rho} < \hat{\rho^*}$. *Otherwise, the long-term self always commits to using push notifications.*

Proof. See Appendix H. ■

consumer self-control problems typically using the hyperbolic discounting framework. These include for example the design of pricing contracts (Della Vigna and Malmendier, 2006), product and size decisions (Jain, 2012a), or sales agent contracts (Jain, 2012b).

The long-term self prefers the short-term self to check less often as it cares less about the immediate anxiety of delaying checking. If push notifications lead to more frequent checking, its benefit of reducing the anxiety cost may not compensate for the increased costs of checking. In the extreme case of $\hat{\rho} \to 0$, the long-term self does not care about future self's anxiety. The only objective is to minimize checking frequency. Therefore, the long-term self will block notifications if receiving notifications induces her future self to check more often. However, if push notifications reduce checking frequency, the long-term self will welcome that since the consumer can save checking cost while suffering less from anxiety. Proposition 4 implies that if the firm allows consumers to choose a push notification policy, we may expect some sophisticated and unconcerned (i.e., lower $\hat{\rho}$) consumers to block notifications.

Specifically, the threshold $\hat{\rho}$ below which the consumer will block push notifications can be calculated to be $\hat{\rho^*} = \left[\frac{1}{2}(V_I + E_I^2)\lambda t^* - \frac{(n^*-1)}{2}V_I\right]^{-1}(\frac{\lambda}{n^*} - \frac{1}{t^*})$. The first term is the inverse of the difference in the average flow disutility between push and no-push strategies. The second term is the difference in average checking costs. At $\hat{\rho^*}$, the two differences cancel out, and the long-term self is indifferent between blocking or not. In terms of comparative statics, locally we may treat n^* as a constant. $\hat{\rho^*}$ is decreasing in E_I^2, implying that the long-term self is less likely to block if the mean utility is high (and thus the anxiety cost is high under the no-push strategy). Similarly, $\hat{\rho^*}$ is decreasing in V_I when n^* is sufficiently small. This suggests that the long-term self is less likely to block if the variance for the information is higher and when the equilibrium checking is frequent, due to similar reasons above. Finally, $\hat{\rho^*}$ is decreasing in the arrival frequency λ when n^* is sufficiently large: $n^* > 2\sqrt{\frac{2\lambda}{(V_I + E_I^2)\rho}}$. It implies that the long-term self is less likely to block when information arrives frequently while checking is sparse. The reason is that push notification will be less likely to induce more frequent checking.

VII Endogenous price

In the main model, there is no price for adopting the service of the information provider. Although this is consistent with the context of many information providers such as e-mails and social media, many information providers, such as newspapers and cable channels, charge consumers subscription prices. This section allows for the endogenous choice of price by the firm.

Suppose that at the beginning of the game, the firm sets a subscription fee p (per unit of time) for its service. The consumer has to pay the fee to check for information. Otherwise, the consumer gets the outside option with utility normalized to zero. The firm's flow profit is given by $\pi = p + \frac{\pi_c}{t}$ in which t is the average checking time for the consumer, π_c is a parameter for the profit per check, and p is the subscription fee. The consumer's decision is to adopt the firm's information platform if $E[U] \geq p$ in which U is the utility flow under the optimal checking strategy. And the firm's decision is to choose p as well as its information design to maximize π. Notice that if $\pi_c > 1$, the consumer's checking yields more profit for the firm than it costs the consumer, implying that checking is socially optimal. Given that the firm is a monopoly, it will set a subscription fee that equals the expected consumer flow utility to extract all the surplus.

Corollary 3 *With subscription fee, the firm prefers the genuine push strategy or the noisy push to no-push strategy if they lead to more frequent checking.*

The intuition for Corollary 3 follows directly from Corollary 1, and Corollary 2. Compared with the no-push strategy, if the genuine push or noisy push strategy leads to a Pareto improvement without endogenous price, then the firm can charge a higher price *and* make a higher profit from more checking. Comparing the genuine push and noisy push strategies, the same result can hold if consumer checking is socially beneficial. Otherwise, the firm may choose a lower level of noise.

Proposition 5 *With subscription fee and $V_I > E_I^2$, if checking is socially optimal ($\pi_c > 1$), the firm's optimal level of noise k_p^* is the same as the optimal noisy push that maximizes checking frequency, given by k^* in Proposition 2. Otherwise, the firm prefers a smaller yet strictly positive level of noise ($0 < k_p^* \leq k^*$) if the profit per check is not too small $\pi_c > \frac{E_I^2}{V_I}$ and $n_p^* > 1$.*

Proof. See Appendix I. ■

When the consumer checks every notification ($n_p^* = 1$), the optimality of noisy push takes a simple form:

Corollary 4 *If under genuine push strategy the consumer checks every notification, the firm prefers genuine push strategy (i.e. $k_p^* = 0$) if $\pi_c < 1$ and optimal noisy push strategy (i.e. $k_p^* = k^*$) if $\pi_c > 1$.*

Proof. See Appendix I. ■

In deciding whether to add noise to push, the firm faces a trade-off. Adding more noise can lead to higher profits from increased checking, but it will also lower consumer surplus due to higher checking costs. The anxiety cost is also weakly lower under noisy push because the consumer checks slightly more frequently. If checking is socially optimal, the profits from more frequent checking can offset the losses from lower subscription fees due to increased checking costs. If not, the optimal level of noise may be lower but can still be positive as long as the profit per check is not too small.

VIII Conclusion

The consumption of information through smartphone apps, tablets, and other digital media is one of the central aspects of the digital consumer economy. Recent studies have pointed to the consistent and significant increases in the amount of time Americans spent on their mobile devices. A 2019 estimate suggests adults have spent more time on their smartphones (> 3.5 hours a day) than watching TV. Thus user engagement and checking of content is a valuable commodity, and firms would like to design their information revelation policies to maximize the amount of checking.

We study the dynamic design of information notifications by a firm that faces a consumer who has consumption utility as well as disutility for realized uncertainty of the information stock at any point in time. The consumer is uncertain both about the arrival of information as well as the value of the arrived information. The firm can strategically design the presentation of notifications to consumers in order to maximize consumer checking. Push notifications, by definition, resolve the arrival uncertainty faced by consumers and leave behind only the uncertainty in the valuation of the information stock. Despite the fact that push notifications reduce consumer uncertainty, we find that they can lead to more frequent checking as compared to no-push. Push notifications create an endogenous impulse to check and discontinuously increases the anxiety costs. Specifically, push notifications are preferred by the firm when the variance of the information is relatively high compared to the mean valuation, and when the information arrival rates are lower.

The firm has the incentive to strategically add noise to its notification design by mixing phantom pushes along with genuine pushes. Noisy push increases the checking frequency even when the consumer is fully rational. Slower true information arrivals and higher relative variance of the information enhance the firm's ability to introduce noise. We also find that while push notifications enhance consumer welfare compared with the no-push case, the noisy push strategy may lower consumer welfare. In an extension, we show the linkage of consumer self-control problems through a dual-self framework to the incentive of consumers to block notifications.

Our results have broad implications for information consumption. Information providers such as news agencies, email, messaging services, as well as social media, all make a profit based on consumer checking of information. Almost all information providers use push technologies to consumers' smartphones, tablets, and computers. Understanding how consumers react to different push technologies will have implications both for the design of the optimal push strategy as well as its implications for consumer welfare. Platforms such as Google and Apple have incentives to regulate push notifications of information providers. As we have already argued, it is hard to overstate the magnitude of information consumption on smartphones and mobile devices not only in people's personal lives but also on their work productivity. A recent study estimates that employees in the U.S. firms spent five hours per week on non-productive activities on their smartphones, and \$15.5 billion loss of productivity per week, or over \$800 billion per year. Even small changes to checking behavior can have significant productivity implications. Our analysis provides a framework for platforms and policymakers to consider the welfare implications of push technologies.

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A Proofs

A Proof of Lemma 1

The informational utility for time τ (if the consumer checks) is given by $u(\tau) = \sum_{k=1}^{N(\tau)} I_k$. We have the expected utility and its variance are given by

$$\mathrm{E}[u(\tau)] = \mathrm{E}[I_k]\mathrm{E}[N(\tau)] = E_I\lambda\tau \tag{3}$$

and

$$\begin{aligned}\mathrm{Var}[u(\tau)] &= \mathrm{Var}[\sum_{k=1}^{N(\tau)} I_k] \\ &= \mathrm{E}[N(\tau)]\mathrm{Var}[I_k] + (\mathrm{E}[I_k])^2\mathrm{Var}[N(\tau)] \\ &= (V_I + E_I^2)\lambda\tau \end{aligned} \tag{4}$$

Both are linear in τ. Therefore, with disutility flow of $\rho\mathrm{Var}[\sum_{k=1}^{N(\tau)} I_k]$, the consumer is getting linearly increasingly "anxious" about unchecked information. The optimal checking strategy is a constant frequency of checking. Suppose the consumer checks every t units of time. The expected flow utility is given by:

$$\begin{aligned}\mathrm{E}[U(t)] &= \frac{1}{t}\left[\mathrm{E}[u(t)] - 1 - \int_0^t \rho\mathrm{Var}[u(\tau)]d\tau\right] \\ &= E_I\lambda - \frac{1}{t} - \frac{1}{2}\rho(V_I + E_I^2)\lambda t \end{aligned} \tag{5}$$

in which the first term is flow consumption utility of information, the second term is flow cost of checking, and the third term is average disutility of carrying realized uncertainty for the period t. It is maximized when $\frac{1}{t} = \frac{1}{2}\rho(V_I + E_I^2)\lambda t$ or $t^* = \sqrt{\frac{2}{\rho(V_I + E_I^2)\lambda}}$. For the optimal expected utility, we have that $\mathrm{E}[U(t^*)] = E_I\lambda - \sqrt{2\rho(V_I + E_I^2)\lambda}$.

B Proof of Lemma 2

We first show that the optimal checking strategy is to check every n_p^* notifications instead of checking between notifications with positive probability. There is no incentive to check between notifications because i) the level of anxiety ($\rho\mathrm{Var}(u_t)$) is constant for all t between two adjacent notifications, and ii) the information arrival process is Poisson and so the expected time for next arrival is also constant. Therefore, checking at any time after the notification arrival is inferior to checking exactly at the notification arrival.

Formally, let $\mathrm{E}[U(n)]$ as the expected flow utility if the consumer uses a policy of checking every n notifications. Define $n_p^* \in \mathcal{N}_+$ as optimal number of notifications to check that maximizes the utility flow. Notice that the expected waiting time for each arrival follows an exponential distribution and has expectation of $\frac{1}{\lambda}$. The expected utility flow is given by

$$\begin{aligned}\mathrm{E}[U(n)] &= \frac{\lambda}{n}\left[nE_I - 1 - \rho\frac{(n-1)n}{2\lambda}V_I\right] \\ &= E_I\lambda - \frac{\lambda}{n} - \rho\frac{(n-1)}{2}V_I \end{aligned} \tag{6}$$

in which the first term is flow consumption utility of information, the second term is flow cost of checking, and the third term is average disutility of carrying realized uncertainty. Minimizing this expected flow utility we have that $\tilde{n_p} = \arg\min_{n\in\mathcal{R}_+} \frac{\lambda}{n} + \rho\frac{(n-1)}{2}V_I$. By definition any $\tilde{n_p} \in [\lfloor\tilde{n_p}\rfloor, \lceil\tilde{n_p}\rceil]$, where $\lceil\tilde{n_p}\rceil = \lfloor\tilde{n_p}\rfloor + 1$. The consumer has to decide whether to check every $\tilde{n_p} \in \lfloor\tilde{n_p}\rfloor$, or every $\lfloor\tilde{n_p}\rfloor + 1$ notifications. The consumer will choose to check every $\tilde{n_p} \in \lfloor\tilde{n_p}\rfloor$ notifications if $\frac{\lambda}{\lfloor\tilde{n_p}\rfloor} + \rho\frac{\lfloor\tilde{n_p}\rfloor - 1}{2}V_I \leq \frac{\lambda}{\lfloor\tilde{n_p}\rfloor + 1} + \rho\frac{\lfloor\tilde{n_p}\rfloor}{2}V_I$. This implies $n_p^* = \lfloor\tilde{n}_p\rfloor$, if $\tilde{n}_p \leq \sqrt{\lfloor\tilde{n}_p\rfloor(\lfloor\tilde{n}_p\rfloor + 1)}$.

We want to show that the strategy of checking every n_p^* notifications yields higher expected flow utility than potential alternate strategies: i.e., i) the strategy of waiting for τ or ii) a probability mixture over a set of the number of notifications after which the consumer checks.

Consider an alternative checking strategy that checks exactly at the n-th notification with some probability $p < 1$, and checks τ units of time after the n-th notification with probability $1 - p$. The expected utility flow of the latter strategy is given by:

$$\begin{aligned}\mathrm{E}[U(n,\tau)] =& \frac{\frac{n}{\lambda}}{\frac{n}{\lambda} + (1-p)\tau}\mathrm{E}[U(n)] + \frac{(1-p)\tau}{\frac{n}{\lambda} + (1-p)\tau}\frac{[nE_I - 1 - \rho(\frac{n(n-1)}{2\lambda} + \tau)V_I]}{\frac{n}{\lambda} + \tau} \\ <& \frac{\frac{n}{\lambda}}{\frac{n}{\lambda} + (1-p)\tau}\mathrm{E}[U(n)] + \frac{(1-p)\tau}{\frac{n}{\lambda} + (1-p)\tau}\frac{[nE_I - 1 - \rho(\frac{n(n-1)}{2\lambda} + \tau)V_I]}{\frac{n}{\lambda}} \\ <& \frac{\frac{n}{\lambda}}{\frac{n}{\lambda} + (1-p)\tau}\mathrm{E}[U(n)] + \frac{(1-p)\tau}{\frac{n}{\lambda} + (1-p)\tau}\frac{[nE_I - 1 - \rho(\frac{n(n-1)}{2\lambda})V_I]}{\frac{n}{\lambda}} \\ =& \mathrm{E}[U(n)] \leq \mathrm{E}[U(n_p^*)] \quad \text{by definition}\end{aligned}$$

Next it can also be shown that the consumer will not check at a combination of different numbers of notifications, since the average utility flow is just a linear combination of strategies of different n which is lower than the strategy of n_p^* with highest utility flow. Suppose the consumer chooses to alternate between a set of checking strategies ($s \in S$) of checking every n_s notifications with probability p_s. And suppose s^* maximizes the expected utility flow $\mathrm{E}[U(n_{s^*})]$. The utility flow will be given by

$$\begin{aligned}\mathrm{E}[U(S)] =& \sum_{s\in S} p_s \mathrm{E}[U(n_s)] \\ <& \mathrm{E}[U(n_{s^*})] \\ \leq& \mathrm{E}[U(n_p^*)]\end{aligned}$$

the last inequality is given by definition of n_p^*.

C Proof of Proposition 1

We want to identify the parameter range where the consumer checks more frequently under push. The expected time between any two checks under push is given by $\frac{n_p^*}{\lambda}$, where n_p^* is the optimal number of notifications as given in Lemma 2, while the expected time between two checks under no-push is given by $t_n^* = \sqrt{\frac{2}{\rho(V_I + E_I^2)\lambda}}$ as given in Lemma 1.

To find the range first note that if a consumer prefers checking every $m \in mathcalN_+$ notifications to checking every $m+1$ notifications, she will check at most m notifications.[7] The condition for the consumer to prefer to check (at most) every m notification is that:

$$\frac{\lambda}{m} + \rho\frac{m-1}{2}V_I < \frac{\lambda}{m+1} + \rho\frac{m}{2}V_I$$

or $\lambda < \rho V_I \frac{m(m+1)}{2}$.

On the other hand, the condition that the consumer will check more frequently if they check every m notifications, compared with no-push, is as follows:

$$\frac{m}{\lambda} < \sqrt{\frac{2}{\rho(V_I + E_I^2)\lambda}}$$

or $\lambda > \rho(V_I + E_I^2)\frac{m^2}{2}$.

Combining the two conditions, we have that some λ always exists as long as $V_I > mE_I^2$, as desired. Because these conditions hold for every m, (and not $m+1$) they also hold for $n_p^* = \lfloor \tilde{n}_p$. Thus the equilibrium range for which the push strategy dominates for the firm is given by

A special case is when $V_I > E_I^2$, there is always an interval $\lambda \in (\frac{\rho(V_I + E_I^2)}{2}, \rho V_I)$ such that the consumer checking every notification, and the checking frequency is higher under push notification compared with no-push.

[7] This is because the average anxiety cost increases linearly in the number of notifications, while the average checking cost $\frac{\lambda}{m}$ decreases more slowly over m.

D Proof of Corollary 1

To prove that push strategy leads to higher consumer utility, note the following: At any point in time since last check, the push strategy leads to lower expected anxiety cost. Given that, a consumer under push strategy can always use the same checking strategy under no-push strategy to get strictly higher utility. And the consumer's utility using optimal checking strategy under push strategy should be even higher by definition. Notice that under no-push, the expected anxiety cost at time τ (since last check) is given by $\rho Var_n[u(\tau)] = \rho(V_I + E_I^2)\lambda\tau$. Under push strategy, the same expected anxiety cost at time τ (since last check) is given by $\rho Var_p[u(\tau)] = \rho V_I \lambda\tau < \rho Var_n[u(\tau)]$.

E Proof of Proposition 2

Following an analysis similar to Lemma 2, we will first show that the optimal checking behavior can be characterized by checking every n_x^* notifications. The optimal number of notifications is given as follows: Upon receiving some n notifications, the variance of realized uncertainty is given by $\mathrm{Var}[\sum_{j=1}^{N(\tau)} I_j] = \mathrm{E}[N(\tau)]\mathrm{Var}[I_j] + (\mathrm{E}[I_j])^2\mathrm{Var}[N(\tau)] = \frac{n}{1+k}V_I + E_I^2\frac{nk}{(1+k)^2}$, in which the first term comes from the variance in the information itself, and the second term comes from the uncertainty of the number of information arrivals, given by the variance of binomial distribution with n notifications, and the probability $\frac{1}{1+k}$ of them being genuine.

Therefore, the expected utility flow of checking per n notifications is given by

$$\begin{aligned}
\mathrm{E}[U(n)] &= \frac{(k+1)\lambda}{n}\left[\frac{n}{1+k}E_I - 1 - \frac{\rho}{\lambda(k+1)}\sum_{i=0}^{n-1}\{\frac{i}{1+k}V_I + E_I^2\frac{ik}{(1+k)^2}\}\right] \\
&= \lambda E_I - \frac{(k+1)\lambda}{n} - \frac{\rho(n-1)}{2(k+1)}(V_I + E_I^2\frac{k}{1+k}) \qquad (7)
\end{aligned}$$

Maximizing $E[U(n)]$ is as before equivalent to minimizing the sum of the flow costs. Let $\tilde{n}_x = \arg\min_{n\in\mathcal{R}_+}\frac{(k+1)\lambda}{n} + \frac{\rho(n-1)}{2(k+1)}(V_I + E_I^2\frac{k}{1+k})$. Therefore, the optimal checking frequency n_x^* is given by:

$$n_x^* = \arg\min_{n\in\mathcal{N}_+}\frac{(k+1)\lambda}{n} + \frac{\rho(n-1)}{2(k+1)}(V_I + E_I^2\frac{k}{1+k})$$

As a sanity check, when $k \to 0$, $n_x^* = \arg\min_{n\in\mathcal{N}_+}\frac{\lambda}{n} + \frac{\rho(n-1)}{2}V_I$ which is the checking frequency for the genuine push strategy as derived in Lemma 2. When $k \to \infty$, implying that all the push notifications are phantom ones, $n_x^* \to (k+1)\sqrt{\frac{2\lambda}{\rho(V_I + E_I^2)}}$ which is the equivalent to the checking time of $\sqrt{\frac{2}{\rho(V_I + E_I^2)\lambda}}$, the same as the checking frequency under the no-push strategy.

The condition that noisy push strategy leads to a higher checking frequency is given by the following. First, n_x^* is the optimal number of notifications to check under noisy push:

$$\frac{(k+1)\lambda}{n_x^*} + \frac{\rho(n_x^*-1)}{2(k+1)}(V_I + E_I^2\frac{k}{1+k}) \leq \frac{(k+1)\lambda}{n_x^*+1} + \frac{\rho(n_x^*)}{2(k+1)}(V_I + E_I^2\frac{k}{1+k})$$

solving it for λ, we have $\lambda \leq \frac{\rho n_x^*(n_x^*+1)}{2(k+1)^2}(V_I + E_I^2\frac{k}{1+k})$.

Second, the checking frequency under noisy push is higher than no-push:

$$\frac{n_x^*}{(1+k)\lambda} \leq \sqrt{\frac{2}{\rho(V_I+E_I^2)\lambda}}$$

solving it for λ, we have $\lambda \geq \frac{\rho {n_x^*}^2}{2(k+1)^2}(V_I + E_I^2)$.

Lastly, we note that the checking frequency is increasing in k for any given n_x^*. Therefore, the (local) optimal k^* makes the constraint of n binding:

$$\frac{(k+1)^2\lambda}{n(n+1)} = \frac{\rho V_I}{2} + \frac{\rho k E_I^2}{2(k+1)} \tag{8}$$

To show that the local optimal is also the global optimal when n is minimal, we want to show $b = \frac{n}{(k+1)}$ is increasing in k for the equation above (because the checking time is given by $\frac{n}{(k+1)\lambda}$). And because n_x^* is strictly increase in k, it is equivalent to showing $\frac{\partial b}{\partial n} > 0$.

To show that $\frac{\partial b}{\partial n} > 0$, we use implicit function theorem. Let $f = \frac{n\lambda}{(n+1)b^2} - \frac{\rho V_I}{2} - \frac{\rho E_I^2(n-b)}{2n}$. $\frac{\partial b}{\partial n} = -\frac{\partial f/\partial n}{\partial f/\partial b}$. We will show $\partial f/\partial b < 0$ and $\partial f/\partial n > 0$.

$$\begin{aligned}
\partial f/\partial b &= \frac{-2n\lambda}{(n+1)b^3} + \frac{\rho E_I^2}{2n} \\
&= \frac{-2}{b}\left[\frac{\rho V_I}{2} + \frac{\rho E_I^2(n-b)}{2n}\right] + \frac{\rho E_I^2}{2n} \\
&= -\frac{\rho V_I}{b} + \frac{-}{b}\frac{\rho E_I^2(n-b)}{n} + \frac{\rho E_I^2}{2n} \\
&= -\frac{\rho V_I + \rho E_I^2}{b} + \frac{3\rho E_I^2}{2n} \\
&< -\frac{\rho\frac{bn-n+b}{n}E_I^2 + \rho E_I^2}{b} + \frac{3\rho E_I^2}{2n} \\
&= \frac{\rho E_I^2}{nb}\left[-bn + \frac{1}{2}b\right] < 0
\end{aligned}$$

The inequality in the second-to-the-last step comes from $V_I \geq E_I^2\frac{n_x^*-k^*}{1+k^*} = E_I^2\frac{bn-n+b}{n}$.

$$
\begin{aligned}
\partial f/\partial n &= \frac{\lambda}{(n+1)^2 b^2} - \frac{\rho E_I^2 b}{2n^2} \\
&= \frac{1}{(n+1)n}\left[\frac{\rho V_I}{2} + \frac{\rho E_I^2(n-b)}{2n}\right] - \frac{\rho E_I^2 b}{2n^2} \\
&= \frac{\rho}{2(n+1)n^2}\left[nV_I + E_I^2(n-b) - E_I^2 b(n+1)\right] \\
&> \frac{\rho}{2(n+1)n^2}\left[E_I^2(bn-n+2b) + E_I^2(n-b) - E_I^2 b(n+1)\right] \\
&= \frac{\rho E_I^2}{2(n+1)n^2}\left[bn-n+2b+n-b-b(n+1)\right] \\
&= 0
\end{aligned}
$$

The third-to-last step comes from $V_I \geq E_I^2 \frac{n_x^* - k^* + 1}{1+k^*} = E_I^2 \frac{bn-n+2b}{n}$, which holds if $V_I \geq (n_x^*+1)E_I^2$. Therefore, we have $\frac{\partial b}{\partial n} > 0$ for any n smaller than $V_I/E_I^2 - 1$. It then suggests that any $n > n_p^*$ such that $V_I \geq E_I^2 \frac{n_x^* - k^* + 1}{1+k^*}$ cannot be optimal. The global optimal is thus either $n_x^* = n_p^*$ or some n such that $\frac{V_I}{E_I^2} \in (\frac{n_x^* - k^*}{1+k^*}, \frac{n_x^* - k^* + 1}{1+k^*})$.

And with optimal k^*, the condition that $\lambda \leq \frac{\rho n_x^*(n_x^*+1)}{2(k+1)^2}(V_I + E_I^2 \frac{k}{1+k})$ will hold because it takes equality and this also implies that $n_x^* = \lfloor \tilde{n}_x \rfloor$. The only condition we need is $\lambda \geq \frac{\rho {n_x^*}^2}{2(k+1)^2}(V_I + E_I^2)$, or $\frac{\rho n_x^*(n_x^*+1)}{2(k+1)^2}(V_I + E_I^2 \frac{k}{1+k}) \geq \frac{\rho {n_x^*}^2}{2(k+1)^2}(V_I + E_I^2)$, which can be reduced to $V_I \geq E_I^2 \frac{n_x^* - k^*}{1+k^*}$.

Note the constraint above is more likely to hold when k^* (and thus n_x^*) is smaller. To make the condition hold, the smallest n_x^* possible is $n_x^* = n_p^* = \arg\min_{n \in \mathcal{N}_+} \frac{\lambda}{n} + \frac{\rho(n-1)}{2} V_I$, i.e. the number of notifications to check under genuine push notification. And because $\frac{n}{(k+1)}$ is increasing in k (and thus n_x^*), such n_p^* is also the global optimal for the platform.

F Proof of Corollary 2

To prove that the noisy push strategy leads to higher consumer utility than no-push, we show the following: At any point in time since last check, the noisy push strategy leads to on average lower anxiety cost. Given that, a consumer under noisy push strategy can always use the same checking strategy under no-push strategy to get at least as much utility. And the consumer's utility using optimal checking strategy under noisy push strategy should be even higher.

Notice that under no-push, the expected anxiety cost at time τ (since last check) is given by $\rho \mathrm{Var}_n[u(\tau)] = \rho(V_I + E_I^2)\lambda\tau$. Under noisy push strategy, the same expected anxiety cost at time τ (since last check) is given by the following: the expected number of push notifications are

$n(\tau) = (1+k)\lambda\tau$. From the Proof of Proposition 2 in Appendix E, we have that the variance is given by

$$\begin{aligned}\mathrm{Var}_x[u(\tau)] =& \frac{n(\tau)}{1+k}V_I + E_I^2\frac{n(\tau)k}{(1+k)^2} \\ &= V_I\lambda\tau + E_I^2\lambda\tau\frac{k}{1+k} \\ &< (V_I + E_I^2)\lambda\tau\end{aligned}$$

Therefore, we have that the expected anxiety cost at time τ under noisy push satisfies $\rho\mathrm{Var}_x[u(\tau)] < \rho\mathrm{Var}_n[u(\tau)]$, which implies that the anxiety cost is on average higher for no-push. As a sanity check, when $k \to \infty$, the anxiety costs under noisy push and push converge as desired.

G Proof of Proposition 3

Under no-push, by Lemma 1, the average time per check when $E_I \to 0$ is $t_n^* = \sqrt{\frac{2}{\rho V_I \lambda}}$. Suppose that under genuine push notification, the consumer checks every m notifications. We have that $\lambda \in \left[\rho V_I\frac{m(m-1)}{2}, \rho V_I\frac{m(m+1)}{2}\right)$. Clearly, when $\lambda \in \left[\rho V_I\frac{m^2}{2}, \rho V_I\frac{m(m+1)}{2}\right)$, the genuine push strategy leads to more frequent checking than no-push, and vice versa.

Now suppose the information provider commits to a noisy push strategy. The noise rate k for some m is given by the following condition: The provider introduces just enough phantom pushes such that the consumer is indifferent between checking at every m notifications and every $m+1$ notifications. Therefore k is given by

$$\frac{(k+1)\lambda}{m} + \frac{\rho(m-1)}{2(k+1)}V_I = \frac{(k+1)\lambda}{m+1} + \frac{\rho m}{2(k+1)}V_I$$

or $(k+1) = \sqrt{\frac{m(m+1)\rho V_I}{2\lambda}}$.

Under this noisy push strategy, the average time per check is $t_p^* = \frac{m}{(k+1)\lambda} = \sqrt{\frac{2m}{\rho V_I \lambda(m+1)}} = t_n^*\sqrt{\frac{m}{m+1}}$. Therefore, on average, the noisy push strategy can improve checking frequency by $\sqrt{\frac{m+1}{m}} - 1$ for any λ.

A special case is when V_I is sufficiently large, such that under genuine push, the consumer checks at every notification, or $V_I > \frac{\lambda}{\rho}$. Then noisy push can improve the frequency over no-push by at most: $\sqrt{2} - 1 = 41\%$. When m grows larger, the improvement is smaller as expected.

H Proof of Proposition 4

The short-term self will want to check more frequently under push notification if the following conditions hold $V_I > n_p^* E_I^2$, and $\lambda \in \left(\rho(V_I + E_I^2)\frac{n_p^{*2}}{2}, \rho V_I \frac{n_p^*(n_p^*+1)}{2}\right)$, where $n_p^* = \lfloor \tilde{n}_p \rfloor$ and $\tilde{n}_p = \arg\min_{n \in \mathcal{R}_+} \frac{\lambda}{n} + \rho\frac{(n-1)}{2}V_I$.

The utility with no-push is given by

$$E[U(t^*)] = E_I\lambda - \frac{1}{t^*} - \frac{1}{2}\rho(V_I + E_I^2)\lambda t^*$$

in which $t^* = \sqrt{\frac{2}{\rho(V_I + E_I^2)\lambda}}$

And the utility with genuine push is given by

$$E[U(n_p^*)] = E_I\lambda - \frac{\lambda}{n_p^*} - \rho\frac{(n_p^* - 1)}{2}V_I$$

The long-term self takes the checking strategies (t^* and n_p^*) as given but evaluates the utilities using the parameter $\hat{\rho} < \rho$:

$$E[\hat{U}(t^*)] = E_I\lambda - \frac{1}{t^*} - \frac{1}{2}\hat{\rho}(V_I + E_I^2)\lambda t^*$$
$$E[\hat{U}(n_p^*)] = E_I\lambda - \frac{\lambda}{n_p^*} - \hat{\rho}\frac{(n_p^* - 1)}{2}V_I$$

The long-term self prefers no-push (i.e., to block push) if $E[\hat{U}(t^*)] > E[\hat{U}(n_p^*)]$. Note that both $E[\hat{U}(t^*)]$ and $E[\hat{U}(n_p^*)]$ are linear in $\hat{\rho}$. Because we know that in the parameter range specified above, the consumer checks more frequently under push, we have $\frac{1}{t^*} < \frac{\lambda}{n_p^*}$. When $\hat{\rho} = 0$, we have $E[\hat{U}(t^*)] > E[\hat{U}(n^*)]$. At $\hat{\rho} = \rho$, by Corollary 1, we know that $E[\hat{U}(t^*)] < E[\hat{U}(n^*)]$. Given that both terms are decreasing linearly in $\hat{\rho}$, there must be a unique $\hat{\rho^*} \in (0, \rho)$, such that $E[\hat{U}(t^*)] > E[\hat{U}(n^*)]$ when $\hat{\rho} \in (0, \hat{\rho^*})$ and $E[\hat{U}(t^*)] < E[\hat{U}(n^*)]$ when $\hat{\rho} \in (\hat{\rho^*}, \rho)$. The threshold $\hat{\rho^*}$ can be calculated as:

$$\begin{aligned}\hat{\rho^*} &= \left[\frac{1}{2}(V_I + E_I^2)\lambda t^* - \frac{(n^*-1)}{2}V_I\right]^{-1}\left(\frac{\lambda}{n^*} - \frac{1}{t^*}\right)\\ &= \left[\sqrt{\frac{(V_I + E_I^2)\lambda}{2\rho}} - \frac{(n^*-1)}{2}V_I\right]^{-1}\left(\frac{\lambda}{n^*} - \sqrt{\frac{\rho(V_I + E_I^2)\lambda}{2}}\right)\end{aligned}$$

The first term is the inverse of the difference in average flow disutility between push and no-push strategies. The second term is the difference in average checking cost. At $\hat{\rho^*}$, the two differences cancel out and the long-term self is indifferent from blocking or not.

In terms of comparative statics, locally we may treat n^* as a constant. Clearly $\hat{\rho^*}$ is decreasing in E_I^2 because both terms are decreasing in E_I^2. It implies that the long-term self is less likely to block if the mean utility is high (and implying the anxiety cost is high under the no-push strategy). Similarly, $\hat{\rho^*}$ is decreasing in V_I when n^* is sufficiently small: $\sqrt{\frac{\lambda}{2(V_I + E_I^2)\rho}} + 1 > n^*$. For example, one sufficient condition is $n^* = 1$. $\hat{\rho^*}$ is decreasing in λ when n^* is sufficiently large: $n^* > 2\sqrt{\frac{2\lambda}{(V_I + E_I^2)\rho}}$.

I Proof of Proposition 5

With subscription fee, under noisy push with noise level k ($k \leq k^*$), the profit is given by the following:

$$
\begin{aligned}
\pi_{noisy} &= E[U(n,k)] + \frac{(k+1)\pi_c\lambda}{n} \\
&= \lambda E_I - \frac{(k+1)\lambda}{n} - \frac{\rho(n-1)}{2(k+1)}(V_I + E_I^2\frac{k}{1+k}) + \frac{(k+1)\pi_c\lambda}{n}
\end{aligned}
$$

When $k = 0$ we have the profit under genuine push strategy.

Taking the derivative with respect to k, we have that

$$
\frac{\partial \pi_{noisy}}{\partial k} = \frac{(\pi_c - 1)\lambda}{n} + \frac{\rho(n-1)}{2(k+1)^2}(V_I - E_I^2\frac{k-1}{k+1})
$$

Clearly $V_I + E_I^2\frac{k-1}{k+1} > 0$ because $V_I > E_I^2$. If $\pi_c - 1 > 0$, then $\frac{\partial \pi_{noisy}}{\partial k} > 0$. The firm would prefer to increase k as long as $k \leq k^*$. If $\pi_c - 1 < 0$, there may exist an interior $k \leq k^*$ such that $\frac{\partial \pi_{noisy}}{\partial k} = 0$.

Notice that at $k = 0$, we have

$$
\begin{aligned}
\frac{\partial \pi_{noisy}}{\partial k}|_{k=0} &= \frac{(\pi_c - 1)\lambda}{n} + \frac{\rho(n-1)}{2}(V_I - E_I^2) \\
&> \frac{(\pi_c - 1)}{n}\frac{\rho V_I n(n-1)}{2} + \frac{\rho(n-1)}{2}(V_I - E_I^2) \\
&= \frac{\rho(n-1)}{2}[\pi_c V_I - E_I^2]
\end{aligned}
$$

and as long as $\pi_c > \frac{E_I^2}{V_I}$, $\frac{\partial \pi_{noisy}}{\partial k}|_{k=0} > 0$, implying the optimal level of noise is strictly positive.

The special case is when consumer checks every notification ($n = 1$). We have $\frac{\partial \pi_{noisy}}{\partial k} = (\pi_c - 1)$. Therefore, the optimal level of noise is either no-push ($k_p^* = 0$) if $\pi_c < 1$ and optimal noisy push strategy (i.e. $k_p^* = k^*$) if $\pi_c > 1$.